

SENSITIVITY OF THE LIDAR RATIO TO CHANGES IN
SIZE DISTRIBUTION AND INDEX OF REFRACTION

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In order to invert lidar signals to obtain reliable extinction coefficients σ , a relationship between σ and the backscatter coefficient β must be given. These two coefficients are linearly related if the complex index of refraction m , particle shape and particle size distribution N does not change along the path illuminated by the laser beam. This, however, is generally not the case. Thus significant deviations from this linear relationship must sometimes be expected.

The relationship between β and σ has been investigated since 1953¹ and intensively in recent years.²⁻⁴ However, theoretical and experimental work to date has been very limited in terms of size distributions and refractive indices covered. Because of this lack of data it is difficult to know, in a typical lidar situation, if a good relationship between β and σ exists.

This study involves an extensive Mie computation of the lidar ratio $R = \beta/\sigma$ and the sensitivity of R to the changes in a parametric space defined by N and m . The real part of m was varied from 0 to 2, the imaginary part from 0 to ∞ , the mode particle size parameter X_m of N was varied from .03 to 3000 and the geometric standard deviation σ_g or width of N was varied from .03 to 1. The log-normal distribution was used the most; however, calculations have been done on many other types, including experimental distributions. For a given m , an average value of R , \bar{R} , was calculated over a set of N . In this set both the mode X_m and width σ_g were varied over a small range. Also the deviation of R over this set, σ_R , was computed. Thus σ_R gives a indication of relative sensitivity of R in a particular region of this parametric space.

The results of the study indicate that there are large volumes of the parametric space in which significant changes in N have little or no effect on R , that is that $\beta \propto \sigma$ in regions difficult to predict theoretically. For example, Fig. 1a shows 4 size distributions, with the same m corresponding to water in the visible, for which $R = .054$. Clearly R is insensitive to changes in N for this set of distributions. Fig. 1b shows other size distributions, again with the same m , for which small changes in N will change R dramatically. The difference between the distributions in Fig. 1a and Fig. 1b is that in Fig. 1b the X_m are small and/or σ_g are small relative to those in Fig. 1a. Thus there is less averaging of the efficiency factors. Figure 2a shows \bar{R}

plotted as a function of the real part of m ; n with the imaginary part $k = 0$. X_m is varied as indicated. Note that there is a general trend for \bar{R} to increase with particle size and that \bar{R} is increasingly sensitive to changes in n with increasing X_m . Very large values of R are obtained for $n = 1.8 - 2.0$ and large X_m . Also note that for pure water in the visible, where $n = 1.33$, \bar{R} changes little. Figure 2b is a plot of σ_g or deviation of R over the set of N versus n . Note that there is extreme sensitivity of R for $n \approx 1.1$ and $n = 1.8 - 2.0$. It is clear that for $n = 1.33$, R is insensitive to changes in N , especially for X_m varying from 12 - 30. Figures 3a and 3b are the same as 2a and 2b except that \bar{R} and σ_g are now plotted against k or absorption. In Fig 3a, \bar{R} initially decreases with increasing absorption, reaching very small values and then rapidly rises with very large values of k . Fig. 3b shows that the sensitivity of R initially stays constant, or decreases, followed by a maximum, and finally R becomes relatively insensitive at large values of k . The maximums of the curves in Fig. 3b are obviously related to the approach of the minimum values in Fig. 3a. This is caused by the fact that distributions with absorption in this region have the large particles absorbing all of the radiation while the small particles still let significant amounts of the radiation scatter or pass through. The maximums occur when $5kX_m \approx 1$.

Since Mie scattering is exact for spheres and thus includes many resonant effects, it is possible that slight deviations from a sphere may have a large effect on R and the sensitivity of R to changes in N . It is known that the most significant resonant effects come from edge-effect rays or rays that just graze the surface of the particles.⁵ These rays may then travel as surface waves and/or enter the particle with further possible scattering. Thus a calculation was performed that neglected these rays and the results compared with the full Mie theory calculation and experimental values taken from the literature.⁶⁻⁸ Figure 4 shows the results. One can see that, without the edge-rays, \bar{R} is considerably less than that predicted by the full Mie theory. Since the 23 experimental points cluster about the full Mie theory, it can be concluded that, in these experiments, the edge-rays contribute the most to R and thus sensitivity of R to changes in N .

Although only a very small portion of the calculation can be presented here, they are indicative of the general behaviour of R and σ_g .

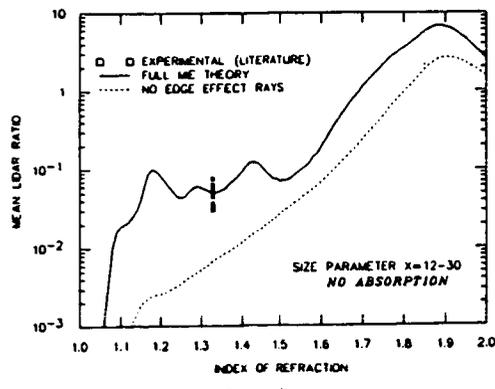
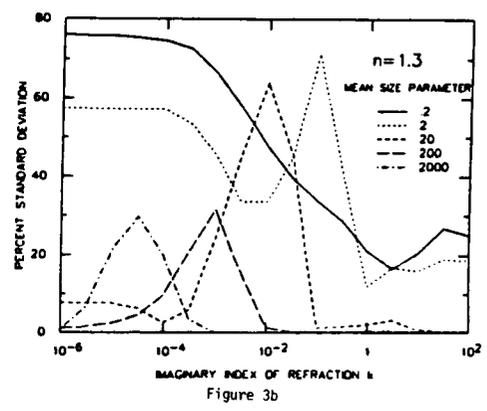
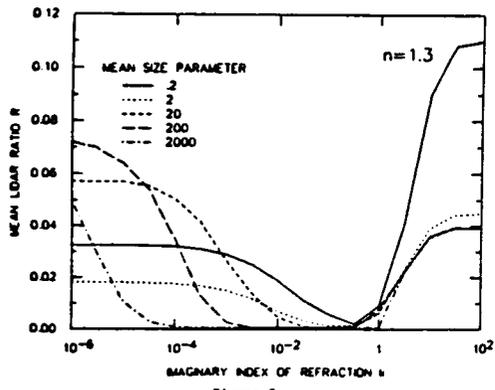
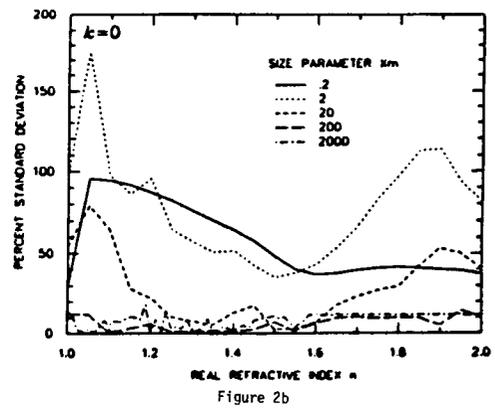
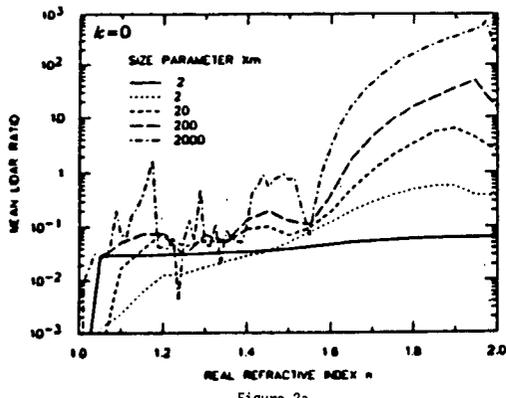
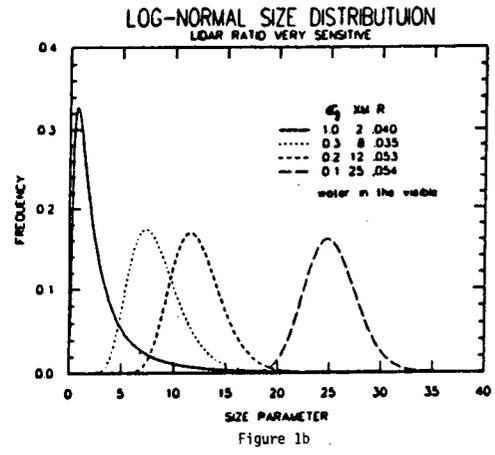
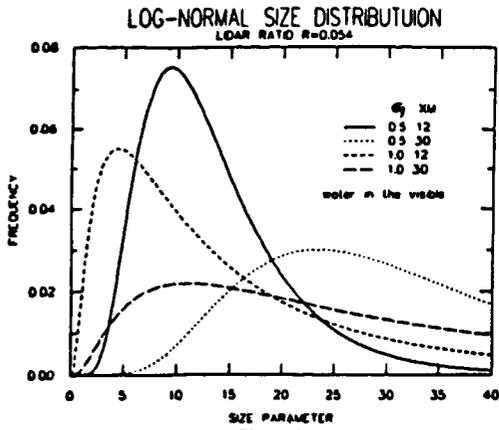
A few of the many conclusions from this study are:

- R is insensitive to changes in N when
 - a) all the particles are non-absorbing and small compared to the wavelength (Rayleigh scattering region)
 - b) for water in the visible and near-infrared, the modal particle size is greater than $1\mu m$
 - c) for water and with wavelengths 6 - $12\mu m$, the modal particle size is greater than $10\mu m$
 - d) $n = 1.5$ and $k \leq 10^{-4}$ and the modal size parameter $X_m \geq 6$
 - e) $2kX_m \geq 1$

- R is sensitive to changes in N when
 - a) for water and with wavelengths .2- $12\mu m$, the modal size parameter $X_m \leq 2$
 - b) the aerosols are of the rural or maritime type⁹ for any wavelength in the visible and infrared
 - c) for water and with the wavelength in the far-infrared, the particle sizes are of the order of several microns (in particular R is many more times more sensitive to size changes at $10.6\mu m$ than at $1.06\mu m^3$)
 - d) $X_m \approx 1$
 - e) relative humidity increases above 70% in hygroscopic aerosols

References

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